

formulation was first given by Grothendieck, extending the work of Hirzebruch to such a relative, functorial setting. Since then several other theorems of this Riemann-Roch type have appeared. Underlying most of these there is a basic structure having to do only with elementary algebra, independent of the geometry. One purpose of this monograph is to describe this algebra independently of any context, so that it can serve axiomatically as the need arises.

Complex Multiplication S. Lang 2012-12-06 The small book by Shimura-Taniyama on the subject of complex multiplication in the higher dimensional case, generalizing in a non-trivial way the method of Deuring for elliptic curves, by reduction mod p . Partly through the work of Shimura himself (cf. [Sh 1] [Sh 2], and [Sh 5]), and some others (Serre, Tate, Kubota, Ribet, Deligne etc.) it is possible today to make a more snappy and extensive presentation of the fundamental results than was possible in 1961. Several persons have found my lecture notes on this subject useful to them, and so I have decided to publish this short book to make them more widely available. Readers acquainted with the standard theory of abelian varieties, and who wish to get rapidly an idea of the fundamental facts of complex multiplication, are advised to look first at the two main theorems, Chapter 3, §6 and Chapter 4, §1, as well as the rest of Chapter 4. The applications of Chapter 6 could also be profitably read early. I am much indebted to N. Schappacher for a careful reading of the manuscript resulting in a number of useful suggestions. S. LANG Contents CHAPTER 1 Analytic Complex Multiplication 4 I. Positive Definite Involutions . . . 6 2. CM Types and Subfields. . . . 8 3. Application to Abelian Manifolds. 4. Construction of Abelian Manifolds with CM 14 21 5. Reflex of a CM Type

Basic Mathematics Serge Lang 1988-01

Real and Functional Analysis Serge Lang 2012-12-06 This book is meant as a text for a first-year graduate course in analysis. In a sense, it covers the same topics as elementary calculus but treats them in a manner suitable for people who will be using it in further mathematical investigations. The organization avoids long chains of logical interdependence, so that chapters are mostly independent. This allows a course to omit material from some chapters without compromising the exposition of material from later chapters.

Cyclotomic Fields I and II Serge Lang 1990-01-01 Kummer's work on cyclotomic fields paved the way for the development of algebraic number theory in general by Dedekind, Weber, Hensel, Hilbert, Takagi, Artin and others. However, the success of this general theory has tended to obscure special facts proved by Kummer about cyclotomic fields which lie deeper than the general theory. For a long period in the 20th century this aspect of Kummer's work seems to have been largely forgotten, except for a few papers, among which are those by Pollaczek [Po], Artin-Hasse [A-H] and Vandiver [Va]. In the mid 1950's, the theory of cyclotomic fields was taken up again by Iwasawa and Leopoldt. Iwasawa viewed cyclotomic fields as being analogues for number fields of the constant field extensions of algebraic geometry, and wrote a great sequence of papers investigating towers of cyclotomic fields, and more generally, Galois extensions of number fields whose Galois group is isomorphic to the additive group of p -adic integers. Leopoldt concentrated on a fixed cyclotomic field, and established various p -adic analogues of the classical complex analytic class number formulas. In particular, this led him to introduce, with Kubota, p -adic analogues of the complex L -functions attached to cyclotomic extensions of the rationals. Finally, in the late 1960's, Iwasawa [Iw 11] made the fundamental discovery that there was a close connection between his work on towers of cyclotomic fields and these p -adic L -functions of Leopoldt - Kubota.

Linear Algebra Serge Lang 1987-01-26 "Linear Algebra" is intended for a one-term course at the junior or senior level. It begins with an exposition of the basic theory of vector spaces and proceeds to explain the fundamental structure theorem for linear maps, including eigenvectors and eigenvalues, quadratic and hermitian forms, diagonalization of symmetric, hermitian, and unitary linear maps and matrices, triangulation, and Jordan canonical form. The book also includes a useful chapter on convex sets and the finite-dimensional Krein-Milman theorem. The presentation is aimed at the student who has already had some exposure to the elementary theory of matrices, determinants and linear maps. However the book is logically self-contained. In this new edition, many parts of the book have been rewritten and reorganized, and new exercises have been added.

Algebra 1990

Linear Algebra Kuldeep Singh 2013-10 "This book is intended for first- and second-year undergraduates arriving with average mathematics grades ... The strength of the text is in the large number of examples and the step-by-step explanation of each topic as it is introduced. It is compiled in a way that allows distance learning, with explicit solutions to all of the set problems freely available online <http://www.oup.co.uk/companion/singh>" -- From preface.

Introduction to Linear Algebra Serge Lang 2012-12-06 This is a short text in linear algebra, intended for a one-term course. In the first chapter, Lang discusses the relation between the geometry and the algebra underlying the subject, and gives concrete examples of the notions which appear later in the book. He then starts with a discussion of linear equations, matrices and Gaussian elimination, and proceeds to discuss vector spaces, linear maps, scalar products, determinants, and eigenvalues. The book contains a large number of exercises, some of the routine computational type, while others are conceptual.

Geometry Serge Lang 2013-04-17 At last: geometry in an exemplary, accessible and attractive form! The authors emphasise both the intellectually stimulating parts of geometry and routine arguments or computations in concrete or classical cases, as well as practical and physical applications. They also show students the fundamental concepts and the difference between important results and minor technical routines. Altogether, the text presents a coherent high school curriculum for the geometry course, naturally backed by numerous examples and exercises.

Algebra Serge Lang 2005-06-21 This book is intended as a basic text for a one year course in algebra at the graduate level or as a useful reference for mathematicians and professionals who use higher-level algebra.

This book successfully addresses all of the basic concepts of algebra. For the new edition, the author has added exercises and made numerous corrections to the text. From MathSciNet's review of the first edition: "The author has an impressive knack for presenting the important and interesting ideas of algebra in just the "right" way, and he never gets bogged down in the dry formalism which pervades some parts of algebra." **Fundamentals of Diophantine Geometry** S. Lang 2013-06-29 Diophantine problems represent some of the strongest aesthetic attractions to algebraic geometry. They consist in giving criteria for the existence of solutions of algebraic equations in rings and fields, and eventually for the number of such solutions. The fundamental ring of interest is the ring of ordinary integers \mathbb{Z} , and the fundamental field of interest is the field \mathbb{Q} of rational numbers. One discovers rapidly that to have all the technical freedom needed in handling general problems, one must consider rings and fields of finite type over the integers and rationals. Furthermore, one is led to consider also finite fields, p -adic fields (including the real and complex numbers) as representing a localization of the problems under consideration. We shall deal with global problems, all of which will be of a qualitative nature. On the one hand we have curves defined over say the rational numbers. If the curve is affine one may ask for its points in \mathbb{Z} , and thanks to Siegel, one can classify all curves which have infinitely many integral points. This problem is treated in Chapter VII. One may ask also for those which have infinitely many rational points, and for this, there is only Mordell's conjecture that if the genus is ≥ 2 , then there is only a finite number of rational points.

Introduction to Algebraic Geometry Serge Lang 2019-03-20 Rapid, concise, self-contained introduction assumes only familiarity with elementary algebra. Subjects include algebraic varieties; products, projections, and correspondences; normal varieties; differential forms; theory of simple points; algebraic groups; more. 1958 edition.

Algebra Serge Lang 2012-11-10 This book is intended as a basic text for a one year course in algebra at the graduate level or as a useful reference for mathematicians and professionals who use higher-level algebra. This book successfully addresses all of the basic concepts of algebra. For the new edition, the author has added exercises and made numerous corrections to the text. From MathSciNet's review of the first edition: "The author has an impressive knack for presenting the important and interesting ideas of algebra in just the "right" way, and he never gets bogged down in the dry formalism which pervades some parts of algebra."

Real Analysis Serge Lang 1983

Linear Algebra and Optimization for Machine Learning Charu C. Aggarwal 2020-05-13 This textbook introduces linear algebra and optimization in the context of machine learning. Examples and exercises are provided throughout this text book together with access to a solution's manual. This textbook targets graduate level students and professors in computer science, mathematics and data science. Advanced undergraduate students can also use this textbook. The chapters for this textbook are organized as follows: 1. Linear algebra and its applications: The chapters focus on the basics of linear algebra together with their common applications to singular value decomposition, matrix factorization, similarity matrices (kernel methods), and graph analysis. Numerous machine learning applications have been used as examples, such as spectral clustering, kernel-based classification, and outlier detection. The tight integration of linear algebra methods with examples from machine learning differentiates this book from generic volumes on linear algebra. The focus is clearly on the most relevant aspects of linear algebra for machine learning and to teach readers how to apply these concepts. 2. Optimization and its applications: Much of machine learning is posed as an optimization problem in which we try to maximize the accuracy of regression and classification models. The "parent problem" of optimization-centric machine learning is least-squares regression. Interestingly, this problem arises in both linear algebra and optimization, and is one of the key connecting problems of the two fields. Least-squares regression is also the starting point for support vector machines, logistic regression, and recommender systems. Furthermore, the methods for dimensionality reduction and matrix factorization also require the development of optimization methods. A general view of optimization in computational graphs is discussed together with its applications to back propagation in neural networks. A frequent challenge faced by beginners in machine learning is the extensive background required in linear algebra and optimization. One problem is that the existing linear algebra and optimization courses are not specific to machine learning; therefore, one would typically have to complete more course material than is necessary to pick up machine learning. Furthermore, certain types of ideas and tricks from optimization and linear algebra recur more frequently in machine learning than other application-centric settings. Therefore, there is significant value in developing a view of linear algebra and optimization that is better suited to the specific perspective of machine learning.

Challenges Serge Lang 2012-12-06 This collection, based on several of Lang's "Files", deals with the area where the worlds of science and academia meet those of journalism and politics: social organisation, government, and the roles that education and journalism play in shaping opinions. In discussing specific cases in which he became involved, Lang addresses general questions of standards: standards of journalism, discourse, and of science. Recurring questions concern how people process information and misinformation; inhibition of critical thinking and the role of education; how to make corrections, and how attempts at corrections are sometimes obstructed; the extent to which we submit to authority, and whether we can hold the authorities accountable; the competence of so-called experts; and the use of editorial and academic power to suppress or marginalize ideas, evidence, or data that do not fit the tenets of certain establishments. By treating case studies and providing extensive documentation, Lang challenges some individuals and establishments to reconsider the ways they exercise their official or professional responsibilities.

Algebraic Number Theory Ian Stewart 1979-05-31 The title of this book may be read in two ways. One is 'algebraic number-theory', that is, the theory of numbers viewed algebraically; the other, 'algebraic-number theory', the study of algebraic numbers. Both readings are compatible with our aims, and both are perhaps misleading. Misleading, because a proper coverage of either topic would require more space than is available, and demand more of the reader than we wish to; compatible, because our aim is to illustrate how some of the basic notions of the theory of algebraic numbers may be applied to problems in number theory. Algebra is an easy subject to compartmentalize, with topics such as 'groups', 'rings' or 'modules' being taught in comparative isolation. Many students view it this way. While it would be easy to exaggerate this tendency, it is not an especially desirable one. The leading mathematicians of the nineteenth and early twentieth centuries developed and used most of the basic results and techniques of linear algebra for perhaps a hundred years, without ever defining an abstract vector space: nor is there anything to suggest that they suffered thereby. This historical fact may indicate that abstraction is not always as necessary as one commonly imagines; on the other hand the axiomatization of mathematics has led to enormous organizational and conceptual gains.

SI2(r) Serge Lang 1985-08-01

Algebra: Chapter 0 Paolo Aluffi 2009 Algebra: Chapter 0 is a self-contained introduction to the main topics of algebra, suitable for a first sequence on the subject at the beginning graduate or upper undergraduate level. The primary distinguishing feature of the book, compared to standard textbooks in algebra, is the early introduction of categories, used as a unifying theme in the presentation of the main topics. A second feature consists of an emphasis on homological algebra: basic notions on complexes are presented as soon as modules have been introduced, and an extensive last chapter on homological algebra can form the basis for a follow-up introductory course on the subject. Approximately 1,000 exercises both provide adequate practice to consolidate the understanding of the main body of the text and offer the opportunity to explore many other topics, including applications to number theory and algebraic geometry. This will allow instructors to adapt the textbook to their specific choice of topics and provide the independent reader with a richer exposure to algebra. Many exercises include substantial hints, and navigation of the topics is facilitated by an extensive index and by hundreds of cross-references.

A Book of Abstract Algebra Charles C Pinter 2010-01-14 Accessible but rigorous, this outstanding text encompasses all of the topics covered by a typical course in elementary abstract algebra. Its easy-to-read treatment offers an intuitive approach, featuring informal discussions followed by thematically arranged exercises. This second edition features additional exercises to improve student familiarity with applications. 1990 edition.

Introduction to Algebraic Geometry Serge Lang 2019-03-20 Author Serge Lang defines algebraic geometry as the study of systems of algebraic equations in several variables and of the structure that one can give to the solutions of such equations. The study can be carried out in four ways: analytical, topological, algebraico-geometric, and arithmetic. This volume offers a rapid, concise, and self-contained introductory approach to the algebraic aspects of the third method, the algebraico-geometric. The treatment assumes only familiarity with elementary algebra up to the level of Galois theory. Starting with an opening chapter on the general theory of places, the author advances to examinations of algebraic varieties, the absolute theory of varieties, and products, projections, and correspondences. Subsequent chapters explore normal varieties, divisors and linear systems, differential forms, the theory of simple points, and algebraic groups, concluding with a focus on the Riemann-Roch theorem. All the theorems of a general nature related to the foundations of the theory of algebraic groups are featured.

Introduction to Algebraic and Abelian Functions Serge Lang 2012-12-06 Introduction to Algebraic and Abelian Functions is a self-contained presentation of a fundamental subject in algebraic geometry and number theory. For this revised edition, the material on theta functions has been expanded, and the example of the Fermat curves is carried throughout the text. This volume is geared toward a second-year graduate course, but it leads naturally to the study of more advanced books listed in the bibliography.

Abelian Varieties Serge Lang 2019-02-13 A basic advanced text in its field, this monograph for undergraduates and graduate students in mathematics requires some background in elementary qualitative algebraic geometry and the elementary theory of algebraic groups. 1959 edition.

Basic Algebra I Nathan Jacobson 2012-12-11 A classic text and standard reference for a generation, this volume covers all undergraduate algebra topics, including groups, rings, modules, Galois theory, polynomials, linear algebra, and associative algebra. 1985 edition.